Examples 4 in Basic Geometry

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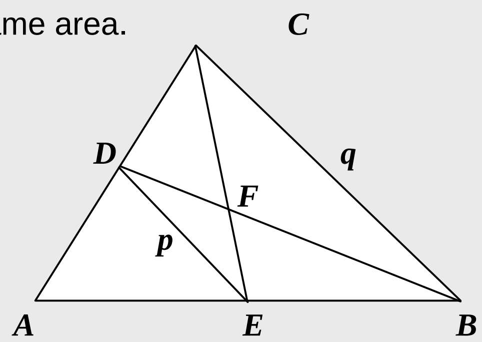
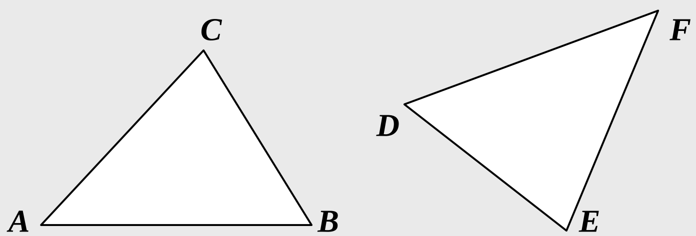
Examples 4

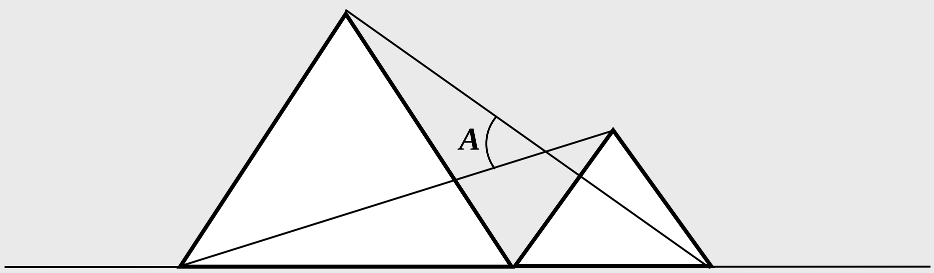
0. Assuming $q = 2p$, and p and q are parallel to each other, find all the triangle that have the same area.



1. Find the conditions under which the two triangles below are identical.



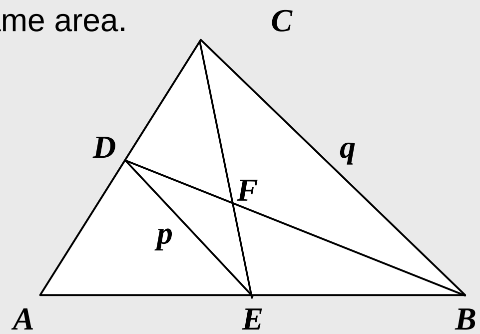
2. Assuming the two triangles below are regular and placed on a line, and meet at one point on the line, find the angle A.



Suggestions or Solutions
To the Problem 0

Assuming in the figure below, $q = 2p$, and p **and** q are parallel to each other, find all the triangles that have the same area.

Fig. 0.0



To begin with, what do we mean by the fact that p is parallel to q?

It means that two triangles AED and ABC are similar to each other.
And if a triangle is similar to another triangle, both have the same set of three angles.

So in this case, we have $\angle DAE = \angle CAB$, $\angle ADE = \angle ACB$, and $\angle AED = \angle ABC$.

And next, $q = 2p$ means that every side in the bigger triangle is twice its corresponding side. What do we mean by the corresponding side though?

The two sides facing the same angle are said to be corresponding to each other. In this case, the side AD faces the angle AED, the side AC faces the angle ABC, and the two angles are the same because both angles are corresponding angles, since p is parallel to q. So AD and AC correspond to each other.

Now, we know every side in the bigger triangle is twice its corresponding side.

So we get $AB = 2AE$, and $AC = 2AD$, as well as $q = 2p$, which means $BC = 2ED$.

That is, the ratio between each pair of corresponding sides is the same.
More specifically, the ratio of every side in ΔABC to its corresponding side in ΔAED is the same, and is 2 in this case.

In other words, every side in ΔABC is twice its corresponding side in ΔAED.
And we can put it this way, too.

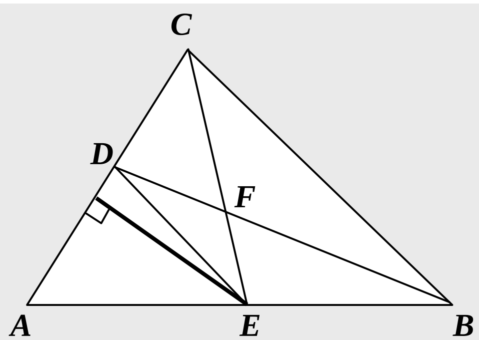
The ratio of every side in ΔAED to its corresponding side in ΔABC is the same, and is $1/2$.
That is to say that every side in ΔAED is half its corresponding side in ΔABC.
So what does the fact above have to do with this problem?

We know that each side in ΔAED is half its corresponding side in ΔABC.
So we can say that D is the midpoint in AC, and E is the midpoint in AB.
That is to say that the area of ΔAED is the same as ΔDEC. Why?

That's because $AD = DC$, and both triangles share the same height.
What then is the height?

The height can be the line segment that connects the point E and a point in AC, and is perpendicular to AC.

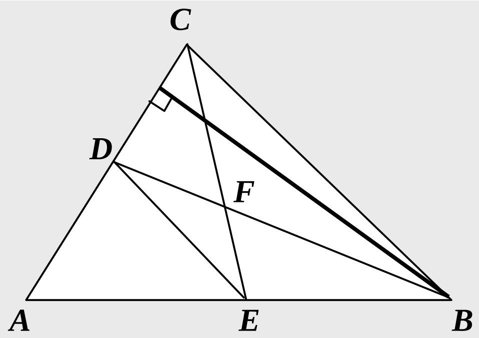
Fig. 0.1



Also, we can say that the area of ΔABD is the same as ΔDBC, because $AD = DC$, and both triangles share the same height. What then is the height?

The height can be the line segment that connects the point ***B*** and a point in ***AC***, and is perpendicular to ***AC***.

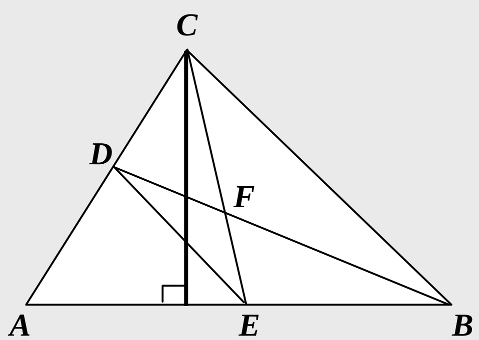
Fig. 0.2



Also, we know that ***E*** is the midpoint in ***AB***.
So we can say that the area of Δ***AEC*** is the same as Δ***BEC***, because ***AE*** = ***EB***, and both triangles share the same height. What then is the height?

The height can be the line segment that connects the point ***C*** and a point in ***AB***, and is perpendicular to ***AB***.

Fig. 0.3



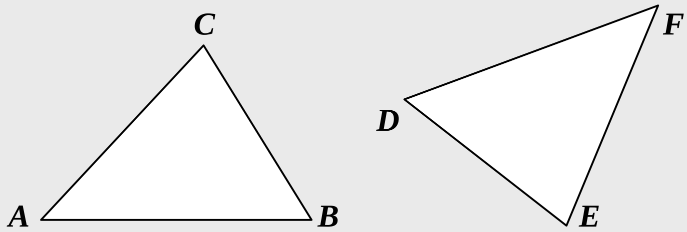
And we can say that the area of Δ***ABC*** is four times the area of Δ***AED***.
Why?

That's because every side in Δ***ABC*** is twice its corresponding side in Δ***AED***.
More specifically, we can notice that the area of Δ***ABC*** is twice the area of Δ***AEC***.
And we know that the area of Δ***AEC*** is twice the area of Δ***AED***.
So we can say that the area of Δ***ABC*** is four times the area of Δ***AED***.

Suggestions or Solutions
To the Problem 1

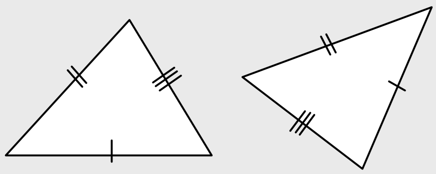
Find the conditions under which the two triangles below are identical.

Fig. 1.0



To begin with, if two triangles have the same set of three sides, the two are identical.
So if $AB = EF$, $BC = DE$, and $CA = DF$, we can say that ΔABC is identical to ΔEFD.

Fig. 1.1



Next, if an angle in one triangle is the same as an angle in the other, and the two sides making the angle in one triangle match the two sides making the same angle in the other triangle, the two triangles are identical.

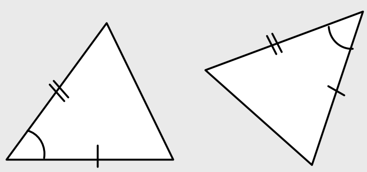
So in either of the three cases below, we can say that ΔABC is identical to ΔEFD.

If $\angle A = \angle F$, $CA = EF$, and $BA = DF$.

If $\angle B = \angle E$, $CB = DE$, and $AB = FE$.

If $\angle C = \angle D$, $AC = FD$, and $BC = ED$.

Fig. 1.2



And next, if two angles in one triangle match two angles in the other, and the side between the two angles in one triangle is the same as the side between the two angles in the other triangle, the two triangles are identical.

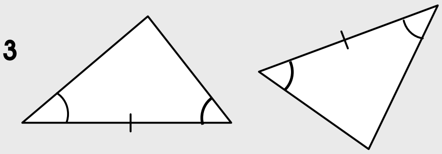
So in either of the three cases below, we can say that $\Delta \boldsymbol{ABC}$ is identical to $\Delta \boldsymbol{EFD}$.

If $\angle \boldsymbol{A} = \angle \boldsymbol{F}$, $\angle \boldsymbol{E} = \angle \boldsymbol{B}$, and $\boldsymbol{AB} = \boldsymbol{FE}$.

If $\angle \boldsymbol{B} = \angle \boldsymbol{E}$, $\angle \boldsymbol{C} = \angle \boldsymbol{D}$, and $\boldsymbol{BC} = \boldsymbol{ED}$.

If $\angle \boldsymbol{C} = \angle \boldsymbol{D}$, $\angle \boldsymbol{A} = \angle \boldsymbol{F}$, and $\boldsymbol{CA} = \boldsymbol{DF}$.

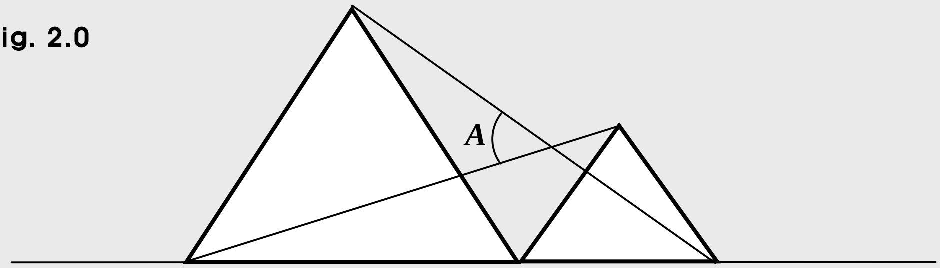
Fig. 1.3



Suggestions or Solutions
To the Problem 2

Assuming the two triangles below are regular and placed on a line, and meet at one point on the line, find the angle ***A*****.**

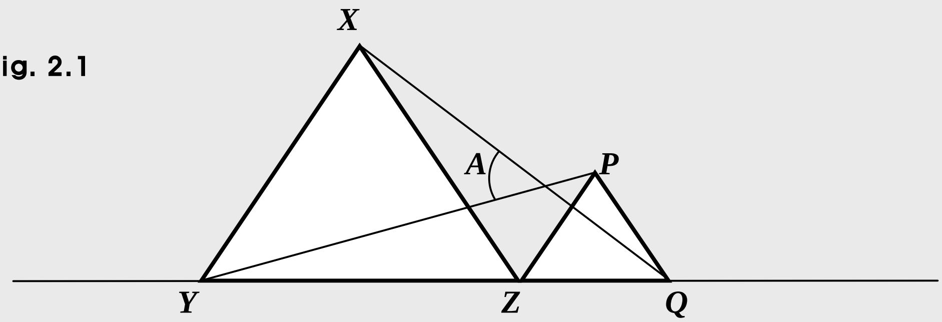
Fig. 2.0



To begin with, in a regular triangle, all the three angles are the same, and so are all the three sides.

Next, adding some labels to the triangles above, we can put them the way below.

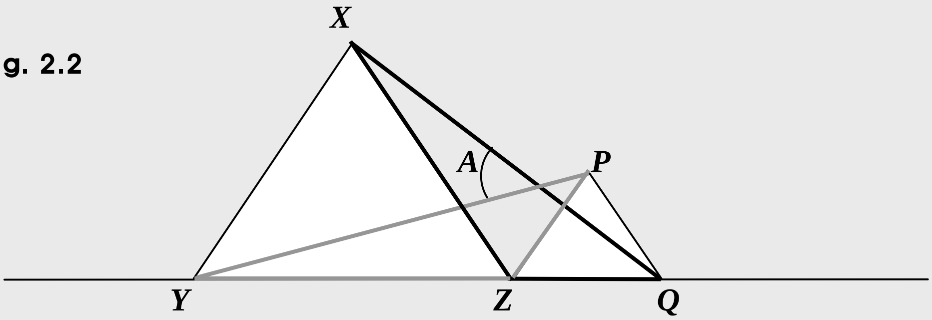
Fig. 2.1



Then, we can notice two triangles identical to each other. One of the two triangles identical is ΔYZP, and the other is ΔXZQ.

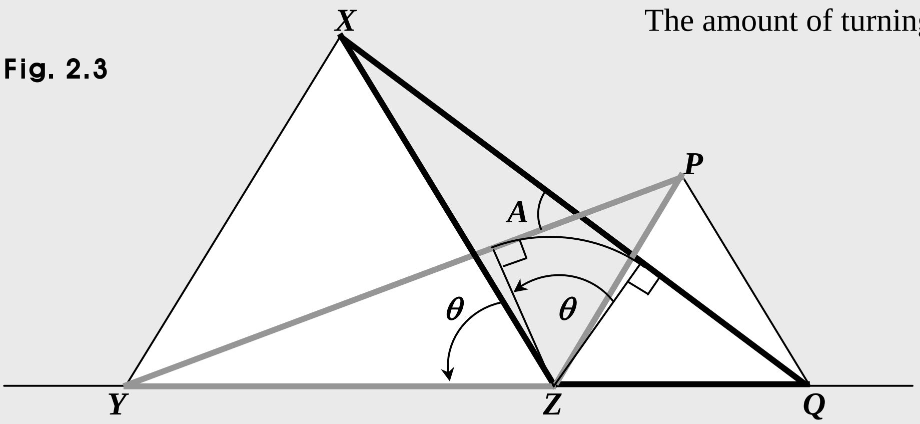
And we can put the two triangles ΔYZP and ΔXZQ the way blow.

Fig. 2.2



And in fact, turning ΔXZQ counterclockwise 60° about the point Z, we get ΔYZP.

Fig. 2.3

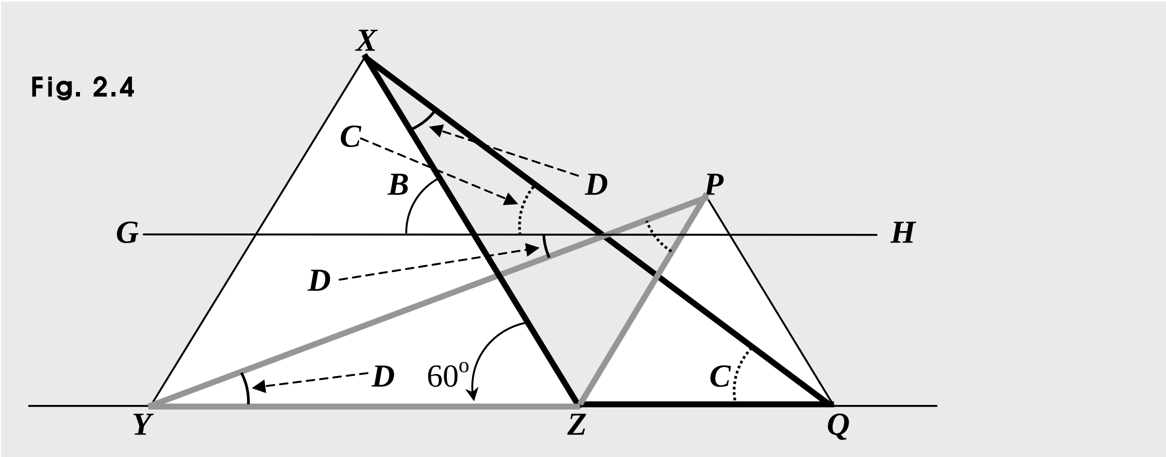


The amount of turning is the angle θ.
So θ is 60°.

That is to say that turning XQ counterclockwise 60° about the point Z, we get YP.

So we can see that the angle A is 60°.

And we can get the same the way below, too.



Assuming ***GH*** is parallel to ***YQ***, we can say that ***B*** = **60°**.

And we get ***C*** + ***D*** = ***B***, because in a triangle, the sum of two internal angles is the same as an external angle supplement to the other internal angle. So in this case, ***C*** + ***D*** is the sum of the two internal angles, and ***B*** is the external angle.

And we know ***B*** = **60°**. So we get ***C*** + ***D*** = **60°**.

Also, we can say that ***C*** + ***D*** = ***A***. So we get ***A*** = **60°**.